Seat No.

* 1 8 3 0 *

Engineering Mathematics - III
(1090)

P. Pages : 4

Time : Three Hours | Max. Marks : 100

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt all questions.
5. Figures to the right indicate full marks.
6. Neat diagrams must be drawn wherever necessary.
7. Use of non-programmable calculator is allowed.
8. Assume suitable data if any.

1. Attempt **any four.** **20**

a) $(D^2 + 2D + 1)y = xe^{-x}\cos x$

b) $(D-1)^4 y = e^x + x^2 + 5\tan^{-1} e$

c) $x^2 \dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} + 2y = e^x$

d) $(D^2 + 2D + 2)\, y = e^{-x}\sec^3 x$ by method of V. P.

e) $\dfrac{dx}{dt} + \dfrac{dy}{dt} + 3x = \sin t$

$\dfrac{dx}{dt} + y - x = \cos t$

f) Find the conditions under which the circuit is overdamped, underdamped & critical damped. Also find critical resistance.

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2. Attempt **any four.** **20**

a) Find an analytic function f(z) whose imaginary part is $e^{-x}(y\sin y + x\cos y)$.

b) Verify whether the function $f(z) = \sinh z$ is an analytic.

c) Use Cauchy's integral formula to evaluate $\oint_C \frac{1}{z}\cos z \, dz$, where 'C' is an ellipse $9x^2 + 4y^2 = 1$.

d) State Residue theorem & use it to evaluate

$$\oint_C \frac{e^{2z} + z^2}{(z-1)^5} dz, \qquad \text{where 'c' is } |z| = 2.$$

e) Find the Bilinear transformation which maps the points 1, i, 2i in z-plane onto the points $-2i$, 0, 1 on w-plane.

f) Find the image & draw the rough sketch of the mapping of the region $1 \le x \le 2,\ 2 \le y \le 3$ under the mapping $w = e^z$.

3. Attempt **any four.** **20**

a) Solve the integral equation $\int_0^\infty f(x)\cos\lambda x \, dx = e^{-\lambda}, \quad (\lambda > o)$.

b) Use Fourier integral to prove that

$$\int_0^\infty \frac{\sin\lambda\pi \sin\lambda x}{1-\lambda^2} d\lambda = \begin{cases} \pi/2 \sin x, & |x| \le \pi \\ 0 \quad , & |x| > \pi \end{cases}$$

c) Find $z\left[\frac{1-\cos\alpha k}{k}\right], \quad k \ge o.$

d) Use partial fraction method to find

$$z^{-1}\left[\frac{2z^2-3z}{z^2-2z-8}\right], \text{(a) } |z|<2 \text{ (b) } |z|>4$$

e) Find Fourier sine transform of $f(x)=\begin{cases}1 & , 0\le x\le 1\\ 0 & , \quad x>1\end{cases}$

hence evaluate $\int_0^{\infty}\frac{\sin^3\lambda}{\lambda}\, d\lambda$.

f) Solve $f(k)-\frac{5}{6}f(k-1)+\frac{1}{6}f(k-2)=\left(\frac{1}{2}\right)^k,\ k\ge o$.

4. Attempt **any four.** **20**

a) Find $L\left[\int_o^t e^{-2t}\frac{\sin^2 t}{t}\,dt\right]$.

b) Find $L\left[(t^2+2t+1)\,H(t+1)+\frac{\sinh t}{t}\,\delta\,(t-3)\right]$.

c) Evaluate $\int_{t=0}^{\infty}\int_{u=0}^{t} e^{-2t}\left(\frac{1-e^{-u}}{u}\right)du\ dt.$

d) Find $L^{-1}\left[\frac{1}{s^3(s^2+1)}\right]$ without convolution theorem.

e) Find $L^{-1}\left[\frac{1}{S}\log\left\{\frac{(S+1)(S-2)}{(S+3)}\right\}\right]$

f) Voltage Ee^{-at} is applied at t = 0, to a L - R circuit. if i = 0 at t = 0, show that $i=\frac{E}{R-aL}\left[e^{-at}-e^{-Rt/L}\right]$ using Laplace transform.

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5. Attempt **any four.** **20**

a) Solve $\frac{\partial^2 u}{\partial t^2} = 4\,\frac{\partial^2 u}{\partial x^2}$ satisfying the conditions $u = \frac{\partial u}{\partial t} = \sin x$ at $t = 0$.

b) Using Green's theorem evaluate $\oint\limits_C \left[x^2y\,dx + x^2dy\right]$ where 'C^{-1}' is the boundary of the triangle whose vertices are (0, 0), (1,0) & (1, 1).

c) Evaluate by using Stoke's theorem

$\oint\limits_C [(x-y-z)i+(y-z-x)j+(z-x-y)k]\cdot d\bar{r}$, over the open surface of

the cylinder $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, bounded by the planes z = h & open at the end z = 0.

d) Use Gauss. Divergence theorem to evaluate

$\iint\limits_S (y^2z^2\,i + z^2x^2 j + x^2y^2\,k)\cdot d\bar{s}$, where S is the upper surface of the

sphere $x^2 + y^2 + z^2 = 9,\ z \geq 0$.

e) Find the work done in moving a charge of $+2C$ from A (6, 0, 0) to B (0, 3,0) along a line AB under electric field given by $\bar{E} = xi - 3yj\ \left(V/_m\right)$.

f) If $\bar{D}$ & $\bar{H}$ satisfy the equations

i) $\nabla \cdot \bar{D} = \rho$ ii) $\nabla \cdot \bar{H} = 0$

iii) $\nabla \times \bar{H} = \frac{1}{C}\left(\frac{\partial \bar{D}}{\partial t} + \rho\bar{V}\right)$ iv) $\nabla \times \bar{D} = -\frac{1}{C}\frac{\partial \bar{H}}{\partial t}$

then show that $\nabla^2\,\bar{D} - \frac{1}{C^2}\frac{\partial^2 \bar{D}}{\partial t^2} = \nabla\rho + \frac{1}{C^2}\frac{\partial}{\partial t}\left(\rho\bar{V}\right)$.

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