Seat No.

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Engineering Mathematics - III
(New) (1090)

P. Pages : 4

Time : Three Hours | Max. Marks : 100

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt all the questions.
5. Figures to the right indicate full marks.
6. Use of non-programmable calculator is allowed.
7. Neat diagram must be drawn wherever necessary.

1. Attempt **any four.** **20**

a) $\left(D^2+2\right)y = x^2e^{3x} - \cos 2x + 3^{-2x}$

b) $\left(D^4+8D^2+16\right)y = \sin^2 x + \sin\left(\log 2\right)$

c) $\left(D^2-1\right)y = \operatorname{sech} x$ by method of V. P.

d) $\dfrac{x^2d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 6y = -x^4 \sin x$

e) $(5+2x)^2\dfrac{d^2y}{dx^2} - 6(5+2x)\dfrac{dy}{dx} + 8y = 6x$

f) The voltage 'V' & the current 'i' at a distance 'x' from the sending end of a transmission line satisfy the equation $-\dfrac{dv}{dx} = Ri$ and $-\dfrac{di}{dx} = Gv$, where R & G are constants. If $V = V_0$ at the sending end x = 0 and v = 0 at the receiving end $x = \ell$ prove that $V = V_0\dfrac{\sinh(n\ell - nx)}{\sinh n\ell}$ and $i = V_0\sqrt{\dfrac{G}{R}}\,\dfrac{\cosh(n\ell - nx)}{\sinh n\ell}$ where $n^2 = RG$.

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2. Attempt **any four.** **20**

a) Determine whether the following function is analytic $f(z)=\frac{x-iy}{x^2+y^2}$.

b) If $v=4xy(x^2-y^2)$, find its harmonic conjugate 'u'. Also find $f(z)=u+iv$ in terms of z.

c) Find bilinear transformation that maps the points $z=1,\ i,\ -1$, from z-plane onto the points $w=0,1,\infty$ on w-plane.

d) Evaluate $\oint_c \frac{\sin^2 z}{(z-\pi/6)^3}\,dz$, where c is $|z|=1$.

e) Evaluate $\int_c e^z \operatorname{cosec} z\,dz$, where 'c' is circle $|z|=4$ by Canchys Residue Theorem.

f) Find the map of circle $|z-i|=1$, under the mapping $w=\frac{1}{z}$ into w-plane.

3. a) Attempt **any two.** **12**

a) Using Fourier Integral representation prove that

$$e^{-x}-e^{-2x}=\frac{6}{\pi}\int_0^\infty \frac{\lambda\sin\lambda x}{(\lambda^2+1)(\lambda^2+4)}\,d\lambda$$

b) Find $Z\left[\frac{1}{k!}\right],\ (k\geq 0)$; hence $Z\left[\frac{1}{(k+1)!}\right]$.

c) Find $Z^{-1}\left[\frac{z^3}{(z-1)\left(z-\frac{1}{2}\right)^2}\right]$ by using Inversion Integral method.

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b) Attempt **any two.** **8**

p) Find f (x) if $\int_0^\infty f(x)\sin\lambda x\, dx = \frac{e^{-a\lambda}}{\lambda}$.

q) Find $Z\left[\left(-\frac{1}{2}\right)^{k+1} + 3\left(\frac{1}{2}\right)^{k-1}\right]$ $(k \geq 0)$

r) Solve $f(k+1) - f(k) = 1, \quad f(0) = 0$.

4. Attempt **any four.** **20**

a) Find $L\left[t\int_0^t e^{-2t}\sin 2t\, dt + 4^{-t}\right]$

b) Find $L\left[e^{-(t-\pi)}\cdot\sin(t-\pi)U(t-\pi) + e^{-t}\delta(t)\right]$

c) Evaluate $\int_0^\infty e^{-2t}\frac{\sinh t}{t}\, dt$ by L. T.

d) Find $L^{-1}\left[\frac{S^2+2}{S(S^2+4)}\right]$.

e) Find $L^{-1}\left[\frac{1}{S(S+2)}\right]$ by convolution theorem.

f) Solve $\frac{dy}{dt} + 3y(t) + 2\int_0^t y(t)dt = t, \quad y(0) = 0$.

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5. Attempt **any two.**

a) i) Solve $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ subject to the conditions

$y(0, t) = 0,\ y(\ell, t) = 0,\ \left.\frac{\partial y}{\partial t}\right]_{t=0} = 0$ and $y(x, 0) = \pi\left(\ell x - x^2\right),\ 0 \le x \le \ell$. **7**

ii) Prove that $\int_C (\bar{a} \times \bar{r}) \cdot d\bar{r} = 2\bar{a} \cdot \iint_S d\bar{s}$. **3**

b) i) Find the work done in moving a particle along the curve $x = a\cos\theta,\ y = a\sin\theta,\ z = b\theta$ from $\theta = \pi/4$ to $\pi/2$, under the field of force $\bar{F} = \left(-3a\sin^2\theta\cos\theta\right)i + a\left(2\sin\theta - 3\sin^3\theta\right)j + (b\sin 2\theta)k$. **5**

ii) Evaluate by using Green's theorem $\oint_C \bar{F} \cdot d\bar{r}$, where $\bar{F} = (\sin z)\,i + (\cos x)\,j + (\sin y)\,k$ and 'c' is the boundary of the rectangle $0 \le x \le \pi,\ 0 \le y \le 1,\ z = 3$. **5**

c) i) Evaluate by using stoke's theorem.

$\oint_C [\,ydx + zdy + xdz\,]$, where 'c' is the intersection of the surfaces $x^2 + y^2 + z^2 = a^2$ and $x + z = a$. **5**

ii) Equation of Helectromagnetic wave theory are given by

i) $\nabla \cdot \bar{H} = 0$ ii) $\nabla \times \bar{H} = \frac{1}{C}\left[\frac{\partial \bar{D}}{\partial t} + \rho\bar{V}\right]$

iii) $\nabla \cdot \bar{D} = \rho$ iv) $\nabla \times \bar{D} = -\frac{1}{C}\frac{\partial \bar{H}}{\partial t}$, '$\rho$' being constant

prove that $\nabla^2 \bar{D} = \frac{1}{C^2}\frac{\partial^2 \bar{D}}{\partial t^2} = \nabla\rho + \frac{1}{C^2}\frac{\partial}{\partial t}\left(\rho\bar{V}\right)$. **5**

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