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# Engineering Mathematics - III
## (144111 / 184111 / 234111)

**P. Pages : 4**

**Time : Three Hours** **Max. Marks : 80**

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Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Figures to right indicate full marks.
5. Use of non programmable calculator is allowed.
6. All questions are compulsory.

### UNIT – I

**1.** Attempt **any two.**

a) i) Solve $(D^2+4D+4)y = e^{-2x}+2^x+3$, where $D \equiv \frac{d}{dx}$. **4**

ii) Solve $(D^2+3D+2)y = \sin(e^x)$, where $D \equiv \frac{d}{dx}$. **4**

b) i) Solve $(D^2+1)y = \text{cosec}x$, where $D \equiv \frac{d}{dx}$, by V.P. method. **4**

ii) Solve $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x^5$. **4**

c) An inductor 0.5 H is connected in series with a resistance of 6Ω and a capacitor of 0.02 farad and a voltage generator having alternating voltage 24sin10t, t>0. Find the charge and current at time t if the charge is zero when the switch is closed at t=0. **8**

### UNIT – II

**2.** Attempt **any two.**

a) i) Prove that the function f(z)=sinhz is analytic. **4**

ii) Determine the analytic function w=u+iv if $V = \log(x^2 + y^2) + x - 2y$. **4**

b) i) Find the bilinear transformation which sends the points 1, i, -1 from z-plane into the points i, 0, -i of the w-plane. **4**

ii) Find the image and draw a rough sketch of the mapping of the region $1 \le x \le 2$ and $2 \le y \le 3$ under the mapping $w = e^z$. **4**

c) i) Evaluate $\oint_C \frac{3z^2 + z}{z^2 - 1} dz$, where C is the circle $|z-1| = 1$, by Cauchy's integral formula. **4**

ii) Evaluate $\oint_C \frac{2z - 1}{z(z+1)(z-3)} dz$, where C is the circle $|z| = 2$, by Cauchy's residue theorem. **4**

## UNIT – III

**3.** Attempt **any two.**

a) i) Find $L\left\{e^{-4t}\int_0^t \frac{\sin 3t}{t} dt\right\}$. **4**

ii) Find $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$ by convolution theorem. **4**

b) i) Find $L\left\{(t^2 + 3t + 2) \cup (t-1) + \frac{\sin t}{t}\delta(t-3)\right\}$. **4**

ii) Find $L^{-1}\left\{\log\left(1 + \frac{1}{s^2}\right)\right\}$. **4**

c) Solve, using Laplace transform **8**

$$\frac{dy}{dt} + 3y + 2\int_0^t y(t)dt = t,$$

given that y(0) = 0.

## UNIT – IV

**4.** Attempt **any two.**

a) i) Find Fourier sine transform of **4**

$$f(x)=\begin{cases}1 & -2\le x\le 0\\ -1 & 0<x\le 2,\end{cases}$$

hence show that $\int_0^{\infty}\frac{\cos 2\lambda-1}{\lambda}\sin 2\lambda\, d\lambda=\frac{-\pi}{2}$.

ii) Find z-transform of $f(K)=\frac{2^K}{K}, K\le 1$. **4**

b) **4**

i) Find z-transform of $f(K)=\begin{cases}2^K, & K\ge 0\\ \left(\frac{1}{3}\right)^K, & K<0.\end{cases}$

ii) Find $z^{-1}\left\{\frac{z^2}{z^2+1}\right\}$, by using inversion integral method. **4**

c) Find Fourier transform of **8**

$$f(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0\ , & |x|>1\end{cases}$$

hence evaluate $\int_0^{\infty}\frac{x\cos x-\sin x}{x^3}\cos\left(\frac{x}{2}\right)dx$.

## UNIT – V

**5.** Attempt **any two.**

a) i) Find the magnitudes of tangential and normal components of acceleration for a particle moving on the curve **4**
$x=t^3-4t,\ y=t^2+4t,\ z=8t^2-3t^3$ at $t=2$.

ii) If $u=x+y+z, v=x+y, w=-2xz-2yz-z^2$. Find the value of **4**
$\nabla u\cdot(\nabla v\times\nabla w)$.

b) i) Find the directional derivative of $\phi=4xz^3-3x^2y^2z$ at the point A(2,-1,2) in the direction AB where B(4,-4,8). **4**

ii) Find the directional derivative of $\phi = x^2 + 2y^2 - 3z^2$ at (1, 2,1) in the direction of tangent to the curve $x = t^2 + t,\ y = 2t,\ z = 2 - t$, at $t = 1$. **4**

c) i) Show that $\bar{F} = (2xz^3 + 6y)i + (6x - 2yz)j + (3x^2z^2 - y^2)k$ is irrotational. Find scalar potential $\phi$ such that $\bar{F} = \nabla\phi$. **4**

ii) Show that $\bar{F} = (x^2 - 2x)i + (y^2z + 2y)j - (yz^2 + 2xz)k$ is solenoidal. Is $\bar{F}$ irrotational ? **4**

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