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# Engineering Mathematics - III
## (1090)

**P. Pages : 3**

**Time : Three Hours** **Max. Marks : 100**

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Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt all the questions.
5. Figures to the right indicate full marks.
6. Use of non - programmable calculator is allowed.

**1.** Solve **any four.** **20**

a) $(D^3+1)y = \sin(2x+3)+\cos 2x$

b) $(D^2-2D+1)y = xe^x \sin x$

c) $(D^2+1)y = \operatorname{cosec} x$

d) $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 3y = x^2 \log x$

e) $(3x+2)^2 \dfrac{d^2y}{dx^2} + 3(3x+2)\dfrac{dy}{dx} - 36_y = 3x^2+4x+1$

f) An uncharged condenser of capacity C is charged by applying an e.m.f. E sin $\frac{t}{\sqrt{LC}}$, through leads of self - inductance L and negligible resistance prove that at time t, the charge on one of the plates is

$$\frac{EC}{2}\left[\sin\frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}}\cos\frac{t}{\sqrt{LC}}\right]$$

**2.** Solve **any four** **20**

a) Determine the analytic functions where real part is $x^3-3xy^2+3x^2-3y^2+1$

b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w - i, o, - i$.

c) Use Cauchy's integral formula to evaluate
$\oint_c \frac{e^{2z}}{(z+1)^4}\, dz$, where c is the circle $|Z| = 2$

d) Evaluate $\oint \frac{\sin 5\ z}{z - \frac{\Pi}{2}}\, dz$, where c is the contour $|Z| = 3$.

e) Evaluate $\int_0^{2\Pi} \frac{d\theta}{3 + 2\cos\theta}$.

f) Find the map of circle $|z - 2i| = 1$, under the mapping $w = \frac{1}{z}$ into $w$ – plane

**3.** a) Solve **any two.** **14**

i) Using Fourier integral representation prove that

$$e^{-x} - e^{-2x} = \frac{6}{\Pi}\int_0^{\infty} \frac{\lambda \sin \lambda x}{\left(\lambda^2+1\right)\left(\lambda^2+4\right)} d\lambda$$

ii) Find the Fourier cosine transform of $e^{-x^2}$

iii) Find $z^{-1}\left(\frac{1}{(z-3)(z-2)}\right) 2 < |z| < 3$

b) Solve **any one** **6**

i) Find f (x) if $\int_0^{\infty} f(x) \sin \lambda x\, dx = \frac{e^{-a\lambda}}{\lambda}$

ii) Obtain f (k) given that

$f(k+1) + \frac{1}{2}\ f(k) = \left(\frac{1}{2}\right)^k,\ k \geq 0,\ f(0) = 0.$

**4.** Solve **any four.** **20**

a) Find $L^{-1}\left\{\tan^{-1}\frac{2}{5}\right\}$

b) $L^{-1}\left\{\frac{s}{(s+4)\left(s^2+9\right)}\right\}$ by convolution theorem.

c) L.T. of $f(t) = \begin{Bmatrix} \cos t, & 0 < t < \Pi \\ \sin t, & t > \Pi \end{Bmatrix}$

d) Find L. T. of $\frac{1-\cos t}{t}$

e) Find L - T of $t^2\ U(t-2) - \cosh t\ \delta(t-4)$

f) Solve the differential equation using L .T. Method

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$$

**5.** Solve **any two**

a) i) Solve the partial differential equation. **7**

$$\frac{\partial^2 y}{\partial t^2} = c^2\ \frac{\partial^2 y}{\partial x^2} \quad \text{where}$$

$y(0, t) = 0$, $y(\ell, t) = 0$, $y(x, 0) = a \sin \frac{\Pi x}{\ell}$,

$0 \le x \le \ell$ ; and $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

ii) Prove that $\int_c (\bar{a} \times \bar{r}) \cdot d\bar{r} = 2\,\bar{a} \cdot \iint_s d\bar{s}$ **3**

where c is the boundary of the surface S.

b) i) Evaluate $\int\int_s \bar{F} \cdot \bar{n}\ d\,s$ over the entire surface s of the region **7**

above x oy plane bounded by the cone $z^2 = x^2 + y^2$ and the plane z = 4, if

$$\bar{F} = (4xz)\hat{i} + (xyz^2)\hat{j} + (3z)\hat{k}$$

ii) For any closed surface enclosing volume v, show that **3**

$$\iint_s \text{curt}\ \bar{F} \cdot \hat{n}\ ds = 0$$

c) i) Evaluate $\oint (x-y)\,dx + xy\ dy$ where c is the triangle with vertices (0, 0), (1, 0), (1, 3) **5**

ii) Maxwell's equations are given by **5**

$\nabla \cdot \bar{E} = 0, \nabla \cdot \bar{H} = 0$

$\nabla \times \bar{E} = \frac{\partial \bar{H}}{\partial t}, \nabla \times \bar{H}\ \frac{\partial \bar{E}}{\partial t}$

show that $\bar{E}$ and $\bar{H}$ satisfy the equation

$$\nabla^2\,\bar{u} = \frac{\partial^2\,\bar{u}}{\partial t^2}$$

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